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带p-Laplacian算子非线性分数阶耦合系统边值问题解的存在性

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摘　要:讨论一类带有p-Laplacian算子非线性分数阶微分方程耦合系统边值问题解的存在性,给出解的存在性条件,利用不动点定理进行讨论.

关键词:分数阶微分方程;p-Laplacian算子;耦合系统;边值问题;不动点定理
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Existence of Solutions to Boundary Value Problem for a Coupled System of Nonlinear Fractional Differential Equations with p-Laplace Operator

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Abstract: In this paper, we study the existence of solutions to boundary value problem for a coupled system of nonlinear fractional differential equations with p-Laplace operator. By the fixed theorem, we obtain some sufficient conditions for the existence of the new results.

key words: fractional differential equations; p-Laplacian operator; coupled system; boundary value problem; fixed point theorem

0　引　言

在过去的几十年,分数阶微分方程在科学工程和技术领域得到了广泛应用,例如物理学,机械学,化学,气象学,生物工程等等[1-4],由于分数阶微分方程非常适合刻画具有遗传和记忆材料的物质和过程,所以,分数阶微分方程在科学和工程技术领域的应用有不可替代的作用.众所周知,多孔介质中的湍流是一个基本的力学问题.为了研究

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这类问题,Leibenson[5] 提出了带 p-Laplacian 算子的整数阶微分方程边值问题

$$(\varphi_p(x'(t)))'=f(t,x(t),x'(t)),t\in(0,1)$$

其中 $\varphi_p(s)=|s|^{p-2}s$ 是 p-Laplacian 算子($p>1$). $\varphi_p(s)$ 的共轭算子是 $\varphi_q(s)$, $q>1$ 且 $\frac{1}{p}+\frac{1}{q}=1$. 随着分数阶微分方程自身理论的发展及其应用,近几年,分数阶微分方程边值问题与 Laplacian 算子的结合开始吸引广大学者的兴趣[6-12].

Bashir Ahmad[13] 等研究了一类非线性分数阶微分方程耦合系统的三点边值问题

$$\begin{cases}D^{\alpha}x(t)+f(t,y(t),D^{p}y(t))=0,\\ D^{\beta}y(t)+g(t,x(t),D^{q}x(t))=0,0<t<1,\\ x(0)=y(0)=0,x(1)=\gamma x(\eta),y(1)=\gamma y(\eta)\end{cases}$$

解得存在性, $1<\alpha,\beta<2,0<p,q<1,\alpha-q>1,\beta-p>1,p,q,\gamma>0,0<\eta<1,\gamma\eta^{\alpha-1}<1,\gamma\eta^{\beta-1}<1,\eta\geqslant0$,其中 D^{α},D^{β} 是标准 Riemann-Liouville 型微分, $f,g:[0,1]\times R\times R\to R$ 连续.

程玲玲[14] 等利用 Schauder 不动点定理研究了带有 p-Laplacian 算子分数阶微分方程耦合系统边值问题

$$\begin{cases}D_{0^+}^{\gamma}\varphi_pD_{0^+}^{\alpha}u(t)=f(t,v(t)),0<t<1,\\ D_{0^+}^{\gamma}\varphi_pD_{0^+}^{\beta}v(t)=f(t,u(t)),0<t<1,\\ u(0)=u(1)=v(0)=v(1)=D_{0^+}^{\alpha}u(0)=D_{0^+}^{\beta}v(0)=0\end{cases}$$

解的存在性,其中 $0<\alpha,\beta\leqslant2,0<\gamma\leqslant1,D_{0^+}^{\alpha},D_{0^+}^{\beta},D_{0^+}^{\gamma}$ 是 Caputo 型微分, $f,g:[0,1]\times R\to R$ 连续.

受以上的启发,本文将研究如下一类带 p-Laplacian 算子非线性分数阶微分方程耦合系统边值问题

$$\begin{cases}D_{0^+}^{\gamma}\varphi_pD_{0^+}^{\alpha}x(t)=f(t,y(t),D_{0^+}^{m}y(t)),0<t<1,\\ D_{0^+}^{\gamma}\varphi_pD_{0^+}^{\beta}y(t)=g(t,x(t),D_{0^+}^{n}x(t)),0<t<1,\\ x(0)=y(0)=0,x(1)=rx(\eta),y(1)=r(\eta),\\ D_{0^+}^{\alpha}x(0)=D_{0^+}^{\beta}y(0)=0\end{cases}\qquad(1)$$

其中, $D_{0^+}^{\alpha},D_{0^+}^{\beta},D_{0^+}^{\gamma}$ 是 *Caputo* 型微分, $0<\gamma<1,1<\alpha,\beta<2,0<p,q<1,\alpha-q>1,\beta-p>1,p,q,r>0,0<\eta<1,r\eta<1,\eta\geqslant0,f,g:[0,1]\times R\to R$ 连续,利用 Schauder 不动点定理,得到解的存在性定理.

1　预备知识

引理 1[14]　设 U 为 Banach 空间 X 的一个有界闭凸子集,如果 $T:U\to U$ 为全连续算子,那么 T 在 U 内至少有一个不动点.

定义 2[15]　连续函数 $u(t):R^{+}\to R$ 的 α 次 Riemann-Liouville 型积分为

$$I_{0^+}^{\alpha}u(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}u(s)\mathrm{d}s$$

其中 $\alpha>0$.

定义 3[15]　连续函数 $u(t):R^{+}\to R$ 的 α 次 Caputo 型微分为

$$D_{0^+}^{\alpha}u(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t(t-s)^{n-\alpha-1}u^{(n)}(s)\mathrm{d}s$$

其中, $\alpha>0,n=[\alpha]+1$.

引理 4[15]　设函数 $u\in C[0,1]$ 有 $\alpha>0$ 阶 Caputo 型微分,则有

$$I_{0^+}^{\alpha}D_{0^+}^{\alpha}x(t)=x(t)+c_0+c_1t+c_2t^2+\cdots+c_{n-1}t^{n-1},$$
$$c_i\in R,i=1,2,3,\cdots,n-1$$

其中 n 是大于等于 α 的最小整数.

2　主要结果

引理 5　设 $f(t)\in C[0,1],1<\alpha\leqslant2,0<r\leqslant1$, 则

$$\begin{cases}D_{0^+}^{r}\varphi_pD_{0^+}^{\alpha}x(t)=f(t),0<t<1,\\ x(0)=0,x(1)=\gamma x(\eta),D_{0^+}^{\alpha}x(0)=0\end{cases}\qquad(2)$$

的唯一解为 $x(t)=\int_0^1G(t,s)\varphi_q(I_{0^+}^{r}f(s))\mathrm{d}s$, 其中

$$G(t,s)=\frac{1}{\Gamma(\alpha)(1-\gamma\eta)}\begin{cases}(t-s)^{\alpha-1}(1-\gamma\eta)+\gamma(\eta-s)^{\alpha-1}t-(1-s)^{\alpha-1}t, & 0\leqslant s\leqslant\min\{t,\eta\}<1;\\ (t-s)^{\alpha-1}(1-\gamma\eta)-(1-s)^{\alpha-1}t, & \eta<s\leqslant t;\\ t[(1-s)^{\alpha-1}-\gamma(\eta-s)^{\alpha-1}], & t<s\leqslant\eta;\\ t(1-s)^{\alpha-1}, & \max\{t,\eta\}<s\leqslant1.\end{cases}\qquad(3)$$

证　由引理 2, 则(2)中的方程可化为 $\varphi_pD_{0^+}^{\alpha}x(t)=I_{0^+}^{r}f(t)+c$. 由边值条件 $D_{0^+}^{\alpha}x(t)=0$ 可得 $c=0$,同理可得

$$x(t)=I_{0^+}^{\alpha}\varphi_qI_{0^+}^{r}f(t)+c_0+c_1t\qquad(4)$$

由边值条件 $x(0)=0,x(1)=\gamma x(\eta)$, 得

$$c_0=0,c_1=\frac{1}{\Gamma(\alpha)(1-\gamma\eta)}(\int_0^{\eta}\gamma(\eta-s)^{\alpha-1}f(s)\mathrm{d}s-\int_0^1(1-s)^{\alpha-1}f(s)\mathrm{d}s)$$

,代入式(4),即得

$$x(t)=\int_0^1G(t,s)\varphi_q(I_{0^+}^{r}f(s))\mathrm{d}s$$

其中

$$G(t,s)=\frac{1}{\Gamma(\alpha)(1-\gamma\eta)}\begin{cases}(t-s)^{\alpha-1}(1-\gamma\eta)+\gamma(\eta-s)^{\alpha-1}t-(1-s)^{\alpha-1}t, & 0\leqslant s\leqslant\min\{t,\eta\}<1;\\(t-s)^{\alpha-1}(1-\gamma\eta)-(1-s)^{\alpha-1}t, & \eta<s\leqslant t;\\t[(1-s)^{\alpha-1}-\gamma(\eta-s)^{\alpha-1}], & t<s\leqslant\eta;\\t(1-s)^{\alpha-1}, & \max\{t,\eta\}<s\leqslant 1.\end{cases}$$

证明完毕.

由引理 5,容易得到:

引理 6　微分方程系统(1)就等价于积分方程系统

$$\begin{cases}x(t)=\int_0^1 G_\alpha(t,s)\varphi_q(I_{0^+}^r f(t,y(t),D_{0^+}^m y(t)))\mathrm{d}s,\\y(t)=\int_0^1 G_\beta(t,s)\varphi_q(I_{0^+}^r g(t,x(t),D_{0^+}^n x(t)))\mathrm{d}s,\end{cases}$$

设 $I=[0,1]$,$X=\{x(t)\mid x(t),D_{0^+}^n x(t)\in C(I)\}$,$Y=\{y(t)\mid y(t),D_{0^+}^m y(t)\in C(I)\}$,分别定义空间 X,Y 上的范数

$$\|x(t)\|_X=\max_{t\in I}|x(t)|+\max_{t\in I}|D_{0^+}^n x(t)|,$$

$$\|y(t)\|_X=\max_{t\in I}|y(t)|+\max_{t\in I}|D_{0^+}^m y(t)|.$$

在 $X\times Y$ 上定义范数 $\|(x,y)\|=\max\{\|x\|_X,\|y\|_Y\}$,则 $(X\times Y,\|\cdot\|)$ 为 Banach 空间.

设 $(x,y)\in X\times Y$,根据引理 6 定义算子 T 如下

$$T(u,v)(t)=\left(\int_0^1 G_\alpha(t,s)\varphi_q(I_{0^+}^r f(t,y(t),D_{0^+}^m y(t)))\mathrm{d}s,\int_0^1 G_\beta(t,s)\varphi_q(I_{0^+}^r g(t,x(t),D_{0^+}^n x(t)))\mathrm{d}s\right)=(T_1y(t),T_2x(t)).$$

我们有如下主要定理:

定理 7　设 $f,g:[0,1]\times R\rightarrow R$ 为连续函数,满足以下条件:

(H1) 存在非负函数 $a_1(t),b_1(t),c_1(t),a_2(t),b_2(t),c_2(t)\in L(0,1)$,使

$$f(t,x,y)\leqslant a_1(t)+b_1(t)|x|^{m_1}+c(t)|y|^{m_2}$$

$$g(t,x,y)\leqslant a_1(t)+b_1(t)|x|^{n_1}+c(t)|y|^{n_2}$$

成立,这里 $0<m_1,m_2,n_1,n_2<1$,则边值问题(1)存在解 (x,y).

证　定义 $U=\{(x(t),y(t))\mid x(t)\in X,y(t)\in Y,\|(x(t),y(t))\|\leqslant d,t\in I\}$,则 U 是 $X\times Y$ 的有界凸闭集,且对任意 $(x,y)\in U$,注意到 $f(t,y(t)),g(t,x(t))$ 是有界的.从而算子 T 在 U 上有定义.令

$$M=\max_{t\in I}\int_0^1|G(t,s)|\mathrm{d}s,$$

$$N=\max\{\|a_i\|_\infty,\|b_i\|_\infty,\|c_i\|_\infty,i=1,2\},$$

$$P=\max\{|x(t)|,|D_{0^+}^m y(t)|,|y(t)|,|D_{0^+}^n x(t)|\},$$

且满足 $M\varphi_q(\frac{1}{\Gamma(r+1)}N(1+P^{n_1}+P^{n_2}))\leqslant d$

下面我分两步证明:

第一步,证明 $T:U\rightarrow U$,设 $(x,y)\in U$,应用条件(H1)及引理 3 得

$$|T_1y(t)|\leqslant\left|\int_0^1 G_\alpha(t,s)\varphi_q(I_{0^+}^r f(s,x(s),x^n(s)))\mathrm{d}s\right|\leqslant\int_0^1\left|G_\alpha(t,s)\times\varphi_q(\frac{1}{\Gamma(r)}\int_0^s(s-\tau)^{r-1}f(\tau,x(\tau),x^n(\tau))\mathrm{d}\tau)\right|\mathrm{d}s\leqslant$$

$$\int_0^1\left|G_\alpha(t,s)\varphi_q(\frac{1}{\Gamma(r)}\int_0^1(s-\tau)^{r-1}(a(\tau)+b(\tau)|x(\tau)|^{n_1}+c(\tau)|D_{0^+}^n x(\tau)|^{n_2})\mathrm{d}\tau)\right|\mathrm{d}s\leqslant$$

$$\int_0^1\left|G_\alpha(t,s)\varphi_q(\frac{1}{\Gamma(r)}\int_0^s(s-\tau)^{r-1}\times N(1+|x(\tau)|^{n_1}+|D_{0^+}^n x(\tau)|^{n_2})\mathrm{d}\tau)\right|\mathrm{d}s\leqslant$$

$$\int_0^1|G_\alpha(t,s)|\varphi_q(\frac{1}{\Gamma(r)}\int_0^s(s-\tau)^{r-1}N(1+P^{n_1}+P^{n_2})\mathrm{d}\tau)\mathrm{d}s\leqslant\int_0^1|G_\alpha(t,s)|\mathrm{d}s\varphi_q(\frac{1}{\Gamma(r+1)}\times N(1+P^{n_1}+P^{n_2}))\leqslant M\varphi_q(\frac{1}{\Gamma(r+1)}\times N(1+P^{n_1}+P^{n_2}))\leqslant d$$

同理可得 $|T_2x(t)|<d$.因而 $\|T(u,v)\|\leqslant d$.又因为 $T_1y(t),T_2x(t)$ 在 I 上连续,所以 $T:U\rightarrow U$.

第二步,证 T 是等度连续的,对任意的 $(x,y)\in U$,f,g 有界,令

$M_1=\max\limits_{t\in I}I_{0^+}^r|f(t,y(t),D_{0^+}^m y(t))|$,$M_2=\max\limits_{t\in I}I_{0^+}^r|g(t,x(t),D_{0^+}^n x(t))|$.

$\forall\varepsilon>0,t_1,t_2\in I,t_1<t_2$,存在 $\delta_1=\min(\delta'_1,\delta''_1,\delta'''_1)$,$\delta_2=\min(\delta'_2,\delta''_2,\delta'''_2)$,

令 $\delta=\min\{\delta_1,\delta_2\}$,下面分三种情况进行讨论:

第一种情况,$0<\eta<t_1<t_2<1$,取 $\delta'_1=\dfrac{\Gamma(\alpha+1)(1-\gamma\eta)}{M_1^{q-1}[\gamma\eta^\alpha+(1-\eta)^\alpha+(\alpha+1)(3-2\gamma\eta)]}\varepsilon$

$$|T_1y(t_1)-T_1y(t_2)|\leqslant M_1^{q-1}(\int_0^\eta|G_\alpha(t_1,s)-$$

$G_\alpha(t_2,s)|\mathrm{d}s+\int_\eta^{t_1}|G_\alpha(t_1,s)-G_\alpha(t_2,s)|)+$
$\int_{t_1}^{t_2}|G_\alpha(t_1,s)-G_\alpha(t_2,s)|\mathrm{d}s+\int_{t_2}^1|G_\alpha(t_1,s)-$
$G_\alpha(t_2,s)|\mathrm{d}s)\leqslant\frac{M_1^{q-1}}{\Gamma(\alpha)(1-\gamma\eta)}(\int_0^\eta|(t_1-s)^{\alpha-1}\times$
$(1-\gamma\eta)+rt_1(\eta-s)^{\alpha-1}-t_1(1-s)^{\alpha-1}-$
$(t_2-s)^{\alpha-1}(1-\gamma\eta)-rt_2(\eta-s)^{\alpha-1}+$
$t_2(1-s)^{\alpha-1}|\mathrm{d}s+\int_\eta^{t_1}|(t_1-s)^{\alpha-1}(1-$
$\gamma\eta)-(1-s)^{\alpha-1}t_1-(t_2-s)^{\alpha-1}(1-\gamma\eta)+$
$t_2(1-s)^{\alpha-1}|\mathrm{d}s+\int_{t_1}^{t_2}|t_1(1-s)^{\alpha-1}-$
$(t_2-s)^{\alpha-1}(1-r\eta)+t_2(1-s)^{\alpha-1}|\mathrm{d}s+$
$\int_{t_2}^1|t_1(1-s)^{\alpha-1}-t_2(1-s)^{\alpha-1}|\mathrm{d}s)\leqslant$
$\frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)}\{|-(t_1-\eta)^\alpha(1-\gamma\eta)+$
$t_1^\alpha(1-\gamma\eta)+rt_1\eta^\alpha+t_1(1-\eta)^\alpha-t_1+$
$(t_2-\eta)^\alpha(1-\gamma\eta)-t_2^\alpha(1-\gamma\eta)-rt_2\eta^\alpha-$
$t_2(1-\eta)^\alpha+t_2|+|(t_1-\eta)^\alpha(1-\gamma\eta)+$
$(1-t_1)^\alpha t_1-(1-\eta)^\alpha t_1+(t_2-t_1)^\alpha(1-$
$\gamma\eta)-(t_2-\eta)^\alpha(1-\gamma\eta)-(1-t_1)^\alpha t_2+$
$(1-\eta)^\alpha t_2|+|-t_1(1-t_2)^\alpha+t_1(1-t_1)^\alpha-$
$(t_2-t_1)^\alpha(1-r\eta)+t_2(1-t_2)\alpha-t_2(1-$
$t_1)^\alpha|+|t_1(1-t_2)^\alpha-t_2(1-t_2)^\alpha|\}\leqslant$
$\frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)}\{|-(t_1-\eta)^\alpha+t_1^\alpha+$
$(t_2-\eta)^\alpha-t_2^\alpha)(1-\gamma\eta)+(r\eta^\alpha+(1-\eta)^\alpha-$
$1)(t_1-t_2)|+|((t_1-\eta)^\alpha-(t_2-\eta)^\alpha+$
$(t_2-t_1)^\alpha)(1-\gamma\eta)+(t_1-t_2)((1-t_1)^\alpha-$
$(1-\eta)^\alpha)|+|(-(1-t_2)^\alpha+(1-t_1)^\alpha)(t_1-t_2)-$
$(t_2-t_1)^\alpha(1-r\eta)|+|(t_1-t_2)(1-t_2)^\alpha|\}\leqslant$
$\frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)}\{[2\alpha(1-\gamma\eta)+\gamma\eta^\alpha+$
$(1-\eta)^\alpha-1]+[(\alpha+1)(1-\gamma\eta)+$
$1]+[(\alpha+1-\gamma\eta)+1]\}(t_2-t_1)\leqslant$
$\frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)}[\gamma\eta^\alpha+(1-\eta)^\alpha+(\alpha+$
$1)(3-2\gamma\eta)](t_2-t_1)\leqslant\varepsilon$

第二种情况, $0<t_1<\eta<t_2<1$,取

$$\delta''_1=\frac{\Gamma(\alpha+1)(1-\gamma\eta)}{M_1^{q-1}[4\alpha+2\alpha\eta\gamma+2\alpha\eta-\gamma\eta+(1-\eta)^\alpha+1]}\varepsilon$$

$|T_1y(t_1)-T_1y(t_2)|\leqslant M_1^{q-1}(\int_0^{t_1}|G_\alpha(t_1,s)-$
$G_\alpha(t_2,s)|\mathrm{d}s+\int_{t_1}^\eta|G_\alpha(t_1,s)-G_\alpha(t_2,s)|)+$
$\int_\eta^{t_2}|G_\alpha(t_1,s)-G_\alpha(t_2,s)|\mathrm{d}s+\int_{t_2}^1|G_\alpha(t_1,s)-$
$G_\alpha(t_2,s)|\mathrm{d}s)\leqslant\frac{M_1^{q-1}}{\Gamma(\alpha)(1-\gamma\eta)}(\int_0^{t_1}|(t_1-s)^{\alpha-1}\times$
$(1-\gamma\eta)+rt_1(\eta-s)^{\alpha-1}-t_1(1-s)^{\alpha-1}-$
$(t_2-s)^{\alpha-1}(1-\gamma\eta)-rt_2(\eta-s)^{\alpha-1}+$
$t_2(1-s)^{\alpha-1}|\mathrm{d}s+\int_{t_1}^\eta|t_1[(1-s)^{\alpha-1}-$
$\gamma(\eta-s)^{\alpha-1}]-(t_2-s)^{\alpha-1}(1-\gamma\eta)-$
$\gamma(\eta-s)^{\alpha-1}t_2+(1-s)^{\alpha-1}t_2|\mathrm{d}s+$
$\int_\eta^{t_2}|t_1(1-s)^{\alpha-1}-(t_2-s)^{\alpha-1}(1-r\eta)+$
$t_2(1-s)^{\alpha-1}|\mathrm{d}s+\int_{t_2}^1|t_1(1-s)^{\alpha-1}-$
$t_2(1-s)^{\alpha-1}|\mathrm{d}s)\leqslant\frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)}\{|t_1{}^\alpha(1-$
$\gamma\eta)-\gamma t_1(\eta-t_1)^\alpha+rt_1\eta^\alpha+t_1(1-t_1)^\alpha-t_1+$
$(t_2-t_1)^\alpha(1-\gamma\eta)-t_2^\alpha(1-\gamma\eta)+\gamma t_2(\eta-t_1)^\alpha-$
$rt_2\eta^\alpha-t_2(1-t_1)^\alpha+t_2|+|t_1[-(1-\eta)^\alpha+$
$(1-t_1)^\alpha-\gamma(\eta-t_1)^\alpha]+(t_2-\eta)^\alpha(1-\gamma\eta)-$
$(t_2-t_1)^\alpha(1-\gamma\eta)-\gamma(\eta-t_1)^\alpha t_2-(1-\eta)^\alpha t_2+$
$(1-t_1)^\alpha t_2|+|-t_1(1-t_2)^\alpha+t_1(1-\eta)^\alpha-$
$(t_2-\eta)^\alpha(1-r\eta)-t_2(1-t_2)^\alpha+t_2(1-\eta)^\alpha|+$
$|t_1(1-t_2)^\alpha-t_2(1-t_2)^\alpha|\}\leqslant\frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)}\times$
$\{|[(t_2-t_1)^\alpha+t_1{}^\alpha-t_2^\alpha](1-\gamma\eta)+\gamma(\eta-t_1)^\alpha\times$
$(t_2-t_1)+r\eta^\alpha(t_1-t_2)+(1-t_1)^\alpha(t_1-t_2)+$
$(t_2-t_1)|+|(t_2+t_1)[-(1-\eta)^\alpha+(1-t_1)^\alpha-$
$\gamma(\eta-t_1)^\alpha]+[(t_2-\eta)^\alpha-(t_2-t_1)^\alpha](1-\gamma\eta)|+$
$|(t_2+t_1)[(1-\eta)^\alpha-(1-t_2)^\alpha]-(t_2-\eta)^\alpha(1-$
$r\eta)|+|(t_1-t_2)(1-t_2)^\alpha|\}\leqslant\frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)}\times$
$\{|[(\alpha+1)(1-\gamma\eta)+\gamma(\eta-t_1)^\alpha-r\eta^\alpha-$
$(1-t_1)^\alpha+1](t_2-t_1)|+|(1+\eta)[-(1-\eta)^\alpha+$
$(1-t_1)^\alpha]+[(t_2-\eta)^\alpha-(t_2-t_1)^\alpha](1-$
$\gamma\eta)|+|(1+\eta)[(1-\eta)^\alpha-(1-t_2)^\alpha]|+$
$|(t_1-t_2)(1-t_2)^\alpha|\}\leqslant\frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)}\times$
$\{|(\alpha+1)(1-\gamma\eta)|+|\alpha(1+\eta)+\alpha(1-$
$\gamma\eta)|+|\alpha(1+\eta)|+|(1-\eta)^\alpha|\}(t_2-t_1)\leqslant$
$\frac{M_1^{q-1}[4\alpha+2\alpha\eta\gamma+2\alpha\eta-\gamma\eta+(1-\eta)^\alpha+1]}{\Gamma(\alpha+1)(1-\gamma\eta)}(t_2-$

$t_1) \leqslant \varepsilon$

第三种情况,$0 < t_1 < t_2 < \eta < 1$,取

$$\delta'''_1 = \frac{\Gamma(\alpha+1)(1-\gamma\eta)}{M_1^{q-1}[(\alpha+1)(1-\gamma\eta)+\eta+2\alpha\eta+\alpha\gamma+\gamma\eta^\alpha+(1-\eta)^\alpha]}\varepsilon$$

$$|T_1y(t_1) - T_1y(t_2)| \leqslant M_1^{q-1}\Big(\int_0^{t_1} | G_\alpha(t_1,s) - G_\alpha(t_2,s) | ds + \int_{t_1}^{t_2} | G_\alpha(t_1,s) - G_\alpha(t_2,s) |\Big) + \int_{t_2}^{\eta} | G_\alpha(t_1,s) - G_\alpha(t_2,s) | ds + \int_{\eta}^{1} | G_\alpha(t_1,s) - G_\alpha(t_2,s) | ds) \leqslant \frac{M_1^{q-1}}{\Gamma(\alpha)(1-\gamma\eta)}\Big(\int_0^{t_1} | (t_1-s)^{\alpha-1} \times (1-\gamma\eta) + \gamma t_1(\eta-s)^{\alpha-1} - t_1(1-s)^{\alpha-1} - (t_2-s)^{\alpha-1}(1-\gamma\eta) - rt_2(\eta-s)^{\alpha-1} + t_2(1-s)^{\alpha-1} | ds + \int_{t_1}^{t_2} | t_1[(1-s)^{\alpha-1} - \gamma(\eta-s)^{\alpha-1}] - (t_2-s)^{\alpha-1}(1-\gamma\eta) - \gamma(\eta-s)^{\alpha-1}t_2 + (1-s)^{\alpha-1}t_2 | ds + \int_{t_2}^{\eta} | t_1[(1-s)^{\alpha-1} - \gamma(\eta-s)^{\alpha-1}] - t_2[(1-s)^{\alpha-1} - \gamma(\eta-s)^{\alpha-1}] | ds + \int_{\eta}^{1} | t_1(1-s)^{\alpha-1} - t_2(1-s)^{\alpha-1} | ds\Big) \leqslant$$

$$\frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)}\{| t_1{}^\alpha(1-\gamma\eta) - rt_1(\eta-t_1)^\alpha + rt_1\eta^\alpha + t_1(1-t_1)^\alpha - t_1 + (t_2-t_1)^\alpha(1-\gamma\eta) - t_2^\alpha(1-\gamma\eta) + rt_2(\eta-t_1)^\alpha - rt_2\eta^\alpha - t_2(1-t_1)^\alpha + t_2 | + | t_1[-(1-t_2)^\alpha + (1-t_1)^\alpha - \gamma(\eta-t_2)^\alpha + \gamma(\eta-t_1)^\alpha] - (t_2-t_1)^\alpha(1-\gamma\eta) + \gamma(\eta-t_2)^\alpha t_2 - \gamma(\eta-t_1)^\alpha t_2 - (1-t_2)^\alpha t_2 + (1-t_1)^\alpha t_2 | + | -t_1(1-\eta)^\alpha + t_1(1-t_2)^\alpha - \gamma t_1(\eta-t_2)^\alpha + t_2(1-\eta)^\alpha - t_2(1-t_2)^\alpha + \gamma t_2(\eta-t_2)^\alpha | + | t_1(1-\eta)^\alpha - t_2(1-\eta)^\alpha |\} \leqslant$$

$$\frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)}\{| [t_1{}^\alpha - t_2^\alpha + (t_2-t_1)^\alpha] \times (1-\gamma\eta) + r(\eta-t_1)^\alpha(t_2-t_1) + r\eta^\alpha(t_1-t_2) + (t_1-t_2)(1-t_1)^\alpha + (t_2-t_1) | + | (t_1+t_2) \times [(1-t_1)^\alpha - t_1(1-t_2)^\alpha] + \gamma[(\eta-t_1)^\alpha + (\eta-t_2)^\alpha](t_1-t_2) | + | (1-\eta)^\alpha(t_2-t_1) + (1-t_2)^\alpha(t_1-t_2) + \gamma(\eta-t_2)^\alpha(t_2-t_1) | + | (1-\eta)^\alpha(t_1-t_2) |\} \leqslant \frac{M_1^{q-1}}{\Gamma(\alpha+1)(1-\gamma\eta)} \times \{| (\alpha+1)(1-\gamma\eta) + \eta | + | 2\alpha\eta + \alpha\gamma | + | \gamma\eta^\alpha | + | (1-\eta)^\alpha |\}(t_2-t_1) \leqslant \varepsilon$$

同理可得 $| T_2u(t_1) - T_2u(t_2) | < \varepsilon$. 所以 TU 是等度连续的,同时又是一致有界的,由 Arzela-Ascoli 定理可得是全连续算子,应用引理 1 知 T 存在不动点,所以(1)有解.

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$$u(1+A)-(1+A)\sum_{i=1}^{M(T)}X_i+\sum_{j=1}^{N(T)}Y_j+\sigma W(T)$$
$$=U(T)+(1+A)\sum_{i=M(T)}^{M(t)}X_i-\sum_{j=N(T)}^{N(t)}Y_j-$$
$$[\sigma W(t)-\sigma W(T)]$$

所以

$$E[e^{-RU(t)}\mid T\leqslant t]P(T\leqslant t)=$$
$$E[e^{-RU(T)-R(S(t)-S(T))}\mid T\leqslant t]P(T\leqslant t)=$$
$$E[e^{-RU(T)}\mid T\leqslant t]P(T\leqslant t)$$

当 $t\to\infty$时,

$$e^{-Ru(1+A)}=E[e^{-RU(T)}\mid T<\infty]\psi(u)+$$
$$\lim_{t\to\infty}E[e^{-RU(t)}\mid T>t]P(T>t)$$

当 $T>t$ 时,$U(t)>0$,从而 $e^{-RU(t)}<1$.

由强大数定理知

$$\lim_{t\to\infty}U(t)=+\infty\quad \text{a.s.}$$

因此由控制收敛定理可得

$$\lim_{t\to\infty}E[e^{-RU(t)}\mid T>t]P(T>t)=0\quad \text{a.s.}$$

所以

$$\psi(u)=\frac{e^{-Ru(1+A)}}{E[\exp(-RU(T))\mid T<\infty]}$$

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