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具分布时滞 BAM 神经网络周期解的存在性和指数稳定性

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摘 要:利用重合度理论中的延拓定理和一些分析技巧,获得了具分布时滞的双向联想记忆(BAM)神经网络模型周期解的存在性、唯一性和全局指数稳定的新结论.

关键词:神经网络;周期解;分布时滞

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The Existence and Global Exponential Stability of Periodic Solutions Neural Networks With Distributed Delays

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Abstract: This paper studies the dynamical behavior of bidirectional associative memory neural networks with distributed delays. Based on the continuation theorem of the coincidence degree theory and analytical technique, we obtain some sufficient conditions ensuring the existence, uniqueness, and global exponential stability of periodic solution.

Key words: neural network; periodic solution; distributed delays

双向联想记忆 BAM 神经网络在自动控制等诸多领域有着广泛的应用.目前已经有许多关于 BAM 神经网络模型的存在性及稳定性的研究成果[1-4].文章利用延拓定理和一些分析技巧,获得了具分布时滞 BAM 神经网络模型周期解的存在性、唯一性和全局指数稳定性的充分条件.这些结果对设计全局指数稳定的 BAM 神经网络具有重要的指导意义.模型如下:

$$\begin{cases} \dot{x}_i(t) = -a_i(t)x_i(t) + \\ \quad \sum_{j=1}^{p} a_{ij}(t)f_j\left(\int_{t-\tau_{ij}}^{t} t_{ij}(t-s)y_j(s)\mathrm{d}s\right) + I_i(t) \\ \dot{y}_j(t) = -b_j(t)y_j(t) + \\ \quad \sum_{i=1}^{n} b_{ji}(t)g_i\left(\int_{t-\sigma_{ji}}^{t} \gamma_{ji}(t-s)x_i(s)\mathrm{d}s\right) + J_j(t) \end{cases} \tag{1}$$

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其中 $i=1,2,\ldots,n;j=1,2,\ldots,p$. 设 $a_i(t)$, $b_j(t)$, $a_{ij}(t)$, $b_{ji}(t)$, $I_i(t)$ 和 $J_j(t)$ 在 $t\in[0,\infty)$ 都是周期为 $\omega>0$ 的连续函数, $a_i(t)$ 和 $b_j(t)$ 处处为正, $f_j(t)$ 和 $g_i(t)$ 都连续. $t_{ij}(t)$ 和 $\gamma_{ji}(t)$ 都分段连续, $t_{ij}\geqslant 0$, $\int_0^\infty t_{ij}(t)\mathrm{d}t=1$, $\int_0^\infty e^{\lambda_0 t}t_{ij}(t)\mathrm{d}t<\infty$, $\gamma_{ji}\geqslant 0$, $\int_0^\infty \gamma_{ji}(t)\mathrm{d}t=1$, $\int_0^\infty e^{\lambda_0 t}\gamma_{ji}(t)\mathrm{d}t<\infty$, $\exists\lambda_0>0$. $C(K)=C([-\sigma,0];R^n)\times C([-\tau,0];R^P)$, 对给定初始值 $\Phi=(\varphi,\psi)\in c(K)$ 其中 $\varphi\in C([-\sigma,0];R^n)$, $\psi\in C([-\tau,0];R^P)$ 系统(1)满足初始条件的解 $(x,y)^{\mathrm{T}}:(0,\infty)\to R^{n+p}$, 是连续可微, $t>0$, $x|_{[-\sigma,0]}=\varphi$, $y|_{[-\tau,0]}=\psi$, 其中 $x:[-\sigma,\infty)\to R^n$ 和 $y:[-\tau,\infty)\to R^p$ 是连续映射, $\bar f=\frac{1}{\omega}\int_0^\omega f(t)\mathrm{d}t$, $[f(t)]^+=\max\limits_{t\in[0,\omega]}\{|f(t)|\}$, $[f(t)]^l=\min\limits_{t\in[0,\omega]}\{|f(t)|\}$, f 为 ω 周期连续函数. $\rho(A)$ 为 $A=(a_{ij})_{n\times n}$ 的普半径. $A\geqslant 0$ 也就是 A 的所有元素都大于或等于 0.

定理 1　假设 $a_i(t)>0$ 和 $b_j(t)>0$, $t\geqslant 0$, $i=1,2,\cdots,n$, $j=1,2,\cdots,p$, 且条件(A1)和(A2)成立.

(A1) 存在非负常数, p_j,q_i,α_j,β_i 使得 $|f_j(u)|\leqslant p_j|u|+\alpha_j$, $|g_i(u)|\leqslant q_i|u|+\beta_i$, 对任意的 $t,u\in R$, $i=1,2,\cdots,n$, $j=1,2,\cdots,p$;

(A2) $\rho(M)<1$, 这里 $M=(m_{ij})_{(n+p)\times(n+p)}$ 且 $m_{ij}=$

$$\begin{cases}0, & 1\leqslant i,j\leqslant n, n+1\leqslant i,j\leqslant n+p\\ \left[\dfrac{|a_{i,j-n}(t)|p_{j-n}}{a_i(t)}\right]^+, & 1\leqslant i\leqslant n, n+1\leqslant j\leqslant n+p,\\ \left[\dfrac{|b_{i-n,j}(t)|q_j}{b_{i-n}(t)}\right]^+, & n+1\leqslant i\leqslant n+p, 1\leqslant j\leqslant n.\end{cases}$$

则系统(1)至少存在一个 ω - 周期解.

证明:设所有连续(可微)的 ω - 周期函数 $u(t)=(x(t),y(t))^{\mathrm{T}}$ 的集合记为 $Z(X)$, $x(t)=(x_1(t),\cdots,x_n(t))^{\mathrm{T}}$ 和 $y(t)=(y_1(t),\cdots,y_p(t))^{\mathrm{T}}$ 定义在 R 上, 且记 $|u|_0=\max\limits_{1\leqslant i\leqslant n,1\leqslant j\leqslant p}\{[x_i(t)]^+,[y_j(t)]^+\}$, $i=1,2,\cdots,n$. 对 X 和 Z 分别赋予范数 $|.|_1$ 和 $|.|_0$, 则 X 和 Z 是 Babach 空间, 对 $u\in X$ 和 $z\in Z$, 令 $(Lu)(t)=\dot u(t)$, $Pu=\frac{1}{w}\int_0^\omega u(t)\mathrm{d}t$, $Qz=\frac{1}{w}\int_0^\omega z(t)\mathrm{d}t$ 和 $(Nu)_i(t)=-a_i(t)x_i(t)+\sum_{j=1}^p a_{ij}(t)f_j(\int_{t-\tau_{ij}}^t t_{ij}(t-s)y_j(s)\mathrm{d}s)+I_i(t)$, $i=1,2,\cdots,n$ 且 $(Nu)_{n+j}(t)=-b_j(t)y_j(t)+\sum_{i=1}^n b_{ji}(t)\times g_i(\int_{t-\sigma_{ji}}^t\gamma_{ji}(t-s)x_i(s)\mathrm{d}s)+J_j(t)$, $j=1,2,\cdots,p$.

$$\mathrm{KerL}=R^{n+p},\mathrm{Im}L=\{z\in Z:\int_0^\omega z(t)\mathrm{d}t=0\},$$

在 Z 中是闭集, $\dim\mathrm{Ker}L=n+p=\mathrm{codimIm}L$, 且 $\mathrm{Im}p=\mathrm{Ker}L$, $\mathrm{Ker}Q=\mathrm{Im}L=\mathrm{Im}(I-Q)$. $K_p:\mathrm{Im}L\to\mathrm{Ker}P\cap\mathrm{Dom}L$ 为 L 的广义逆. 由 $(K_pu)_i(t)=\int_0^t u_i(s)\mathrm{d}s-\frac{1}{\omega}\int_0^\omega\int_0^t u_i(s)\mathrm{d}s\mathrm{d}t$, $u=u(t)\in Z$ 给出, 易证 QN 和 $K_p(I-Q)N$ 连续. 对任何有界开集 $\Omega\subset X$, $K_p(I-Q)N(\bar\Omega)$ 是紧的, 则 $QN(\bar\Omega)$ 有界. 对任何有界开集 $\Omega\subset X$, N 在 $\bar\Omega$ 上是 L - 紧的. 由连续延拓定理可得

$$\begin{cases}\dot x_i(t)=-\lambda a_i(t)x_i(t)+\\ \quad\lambda\sum_{j=1}^p a_{ij}(t)f_j(\int_{t-\tau_{ij}}^t t_{ij}(t-s)y_j(s)\mathrm{d}s)+\\ \quad\lambda I_i(t),\quad i=1,2,\cdots,n;\\ \dot y_j(t)=-\lambda b_j(t)y_i(t)+\\ \quad\lambda\sum_{i=1}^n b_{ji}(t)g_i(\int_{t-\sigma_{ji}}^t\gamma_{ji}(t-s)x_i(s)\mathrm{d}s)+\\ \quad\lambda J_j(t),\quad j=1,2,\cdots,p.\end{cases}\tag{2}$$

设对某个 $\lambda\in(0,1)$, $u=u(t)\in X$ 是系统(1)的一个解, 则 $x_i(t)$ 和 $y_j(t)$ 都连续可微, $\exists t_i,t_j'\in[0,\omega]$ 使 $|x_i(t_i)|=[x_i(t)]^+$ 且 $|y_j(t_j')|=[y_j(t)]^+$. 所有 $\dot x_i(t_i)=\dot y_j(t_j')=0$. 则有

$$\begin{cases}a_i(t_i)x_i(t_i)=\sum_{j=1}^p a_{ij}(t_i)f_j(\int_{t_i-\tau_{ij}}^{t_i}t_{ij}(t_i-\\ \quad s)y_j(s)\mathrm{d}s)+I_i(t_i),\\ b_j(t_j')y_j(t_j')=\sum_{i=1}^n b_{ji}(t_j')g_i(\int_{t_j'-\sigma_{ji}}^{t_j'}\gamma_{ji}(t_j'-\\ \quad s)x_i(s)\mathrm{d}s)+J_j(t_j')\end{cases}\tag{3}$$

由方程(2)可得 $|x_i(t_i)|=\left|\sum_{j=1}^p\frac{a_{ij}(t_i)}{a_i(t_i)}f_j(\int_{t_i-\tau_{ij}}^{t_i}t_{ij}(t_i-s)y_j(s)\mathrm{d}s)+\frac{I_i(t_i)}{a_i(t_i)}\right|\leqslant\sum_{j=1}^p m_{i,n+j}|y_j(t_j')|+D_i$.

其中 $D_i=\sum_{j=1}^p[a_{ij}(t)\alpha_j/a_i(t)]^++[I_i(t)/a_i(t)]^+$, $i=1,2,\cdots,n$.

类似的有 $|y_j(t_j')|\leqslant\sum_{i=1}^n m_{n+j,i}|x_i(t_i)|+D_{j+n}$, $j=1,2,\cdots,p$, $D_{n+j}=\sum_{i=1}^n[b_{ji}(t)\beta_i/b_j(t)]^++$

$[J_j(t)/b_j(t)]^+$ (4)

又$\rho(M)<1$,则$(E-M)^T$和$h=(E-M)^{-1}D\geqslant 0$, $D=(D_1,D_2,\cdots,D_{n+p})^T$. 从式(3),(4)可得

$$[x_i(t)]^+\leqslant h_i \quad [y_j(t)]^+\leqslant h_{n+j}, i=1,2,\cdots,n, j=1,2,\cdots,p \tag{5}$$

显然,$h_i, i=1,2,\cdots,n+p$与λ无关. 由式(2)可得

$$[\dot{x}_i(t)]^+\leqslant \max_{t\in[0,\omega]}[\lambda|a_i(t)||x_i(t)|+\lambda\sum_{j=1}^{p}|a_{ij}(t)||f_i(\int_{t-\tau_{ij}}^{t}t_{ij}(t-s)y_j(s)\mathrm{d}s)|+\lambda|I_i(t)|]\leqslant 2h_i[a_i(t)]^+. \tag{6}$$

同理$[\dot{y}_j(t)]^+\leqslant 2h_{n+j}[b_j(t)]^+$ (7)

令$A=\max\limits_{1\leqslant i\leqslant n,1\leqslant j\leqslant p}\{h_i(1+2[a_i(t)]^+), h_{n+j}(1+2[b_j(t)]^+)\}$. $\exists d>1$使$dh_i>A$. 记$\Omega=\{u\in X; -dh<u(t)<dh\}$. 如果$u=(x_1,\cdots,x_n,y_1,\cdots,y_p)^T\in\partial\Omega\cap\mathrm{Ker}L=\partial\Omega\cap R^{n+p}$,则$u$是常向量其中$|x_i|=dh_i$且$|y_j|=dh_{n+j}$, 令$(QNu)_i=\frac{1}{\omega}\int_0^{\omega}[-a_i(t)x_i+\sum_{j=1}^{p}a_{ij}(t)f_j(\int_{t-\tau_{ij}}^{t}t_{ij}(t-s)y_j(s)\mathrm{d}s)+I_i(t)]t, i=1,2,\cdots,n$

反设存在某个$i\in\{1,2,\cdots,n\}$使得$|(QNu)_i|=0$, $\exists$某个$t^*\in[0,\omega]$, 使得$-a_i(t^*)x_i+\sum_{j=1}^{p}a_{ij}(t^*)f_j(\int_{t^*-\tau_{ij}}^{t^*}t_{ij}(t^*-s)y_j(s)\mathrm{d}s)+I_i(t^*)=0$. 因此$dh_i=|x_i|\leqslant\sum_{j=1}^{p}\frac{|a_{ij}(t^*)|}{a_i(t^*)}|f_j(\int_{t^*-\tau_{ij}}^{t^*}t_{ij}(t^*-s)y_j(s)ds)|+\frac{|I_i(t^*)|}{a_i(t^*)}\leqslant\sum_{j=1}^{p}m_{i,n+j}dh_{n+j}+D_i$.

由于$d>1$和$dh=d(Mh+D)>Mdh+D$,有$dh_i>\sum_{j=1}^{n+p}m_{ij}dh_j+D_i, i=1,2,\cdots,n$. $dh_i<dh_i$矛盾,因此,$|(QNu)_i|>0, i\in\{1,2,\cdots,n\}$.

令$(QNu)_{n+j}=\frac{1}{\omega}\int_0^{\omega}[-b_j(t)y_i+\sum_{i=1}^{n}b_{ji}(t)\times g_i(\int_{t-\sigma_{ji}}^{t}\gamma_{ji}(t-s)x_i(s)\mathrm{d}s)+J_j(t)]\mathrm{d}t, j=1,2,\cdots,p$. 类似的,$|(QNu)_i|>0, i\in\{n+1,n+2,\cdots,n+p\}$. 因此$|(QNu)_i|>0, i=1,2,\cdots,n+p$.

对$u\in\partial\Omega\cap\mathrm{Ker}L$, $QN_u\neq 0$. 定义$\psi:\Omega\cap\mathrm{Ker}L\times[0,1]\to X/\mathrm{Im}L=X^c$, $\psi(u,\mu)=\mu diag(-\bar{a}_1,\cdots,-\bar{a}_n,-\bar{b}_1,\cdots,-\bar{b}_p)u+(1-\mu)QNu$,对$u=(x_1,\cdots,x_n,y_1,\cdots,y_p)^T\in\Omega\cap\mathrm{Ker}L=\Omega\cap R^{n+p}$并$\mu\in[0,1]$. 当$u\in\partial\Omega\cap\mathrm{Ker}L$且$u=(x_1,\cdots,x_n,y_1,\cdots,y_p)^T$, $\mu\in[0,1]$, $u,v\in R^{n+p}$. 其中$|x_i|=\mathrm{d}h_i(i=1,2,\cdots,n)$, $|y_j|=\mathrm{d}h_{n+j}(j=1,2,\cdots,p)$. 因此,

$$|\psi(u,\mu)|_0=\max_{1\leqslant i\leqslant n,1\leqslant j\leqslant p}\{|-\bar{a}_ix_i+(1-\mu)[\frac{1}{\omega}\sum_{j=1}^{p}\int_0^{\omega}a_{ij}(t)f_i(\int_{t-\tau_{ij}}^{t}t_{ij}(t-s)y_j(s)\mathrm{d}s)\mathrm{d}t+\bar{I}_i]||-\bar{b}_jy_j+(1-\mu)[\frac{1}{\omega}\sum_{i=1}^{n}\int_0^{\omega}b_{ji}(t)g_i(\int_{t-\sigma_{ji}}^{t}\gamma_{ji}(t-s)x_i(s)\mathrm{d}s)\mathrm{d}t+\bar{J}_j]|\}$$

下面证明$|\psi(u,\mu)|_0>0$反设$|\psi(u,\mu)|_0=0$,有

$$\begin{cases}-\bar{a}_ix_i+\dfrac{(1-\mu)}{\omega}\sum_{j=1}^{p}\int_0^{\omega}a_{ij}(t)f_j(\int_{t-\tau_{ij}}^{t}t_{ij}(t-s)y_j(s)\mathrm{d}s)\mathrm{d}t+(1-\mu)\bar{I}_i=0, i=1,2\cdots,n\\ -\bar{b}_jx_j+\dfrac{(1-\mu)}{\omega}\int_0^{\omega}\sum_{i=1}^{n}b_{ji}(t)g_i(\int_{t-\sigma_{ji}}^{t}\gamma_{ji}(t-s)x_i(s)\mathrm{d}s)\mathrm{d}t+(1-\mu)\bar{J}_j=0, j=1,2\cdots,p\end{cases} \tag{8}$$

$\exists t^*\in[0,\omega]$使$-a_i(t^*)x_i+(1-\mu)\sum_{j=1}^{p}a_{ij}(t^*)f_j(\int_{t^*-\tau_{ij}}^{t^*}t_{ij}(t^*-s)y_j(s)\mathrm{d}s)+(1-\mu)I_i(t^*)=0$即$dh_i\leqslant(1-\mu)\sum_{j=1}^{p}\frac{|a_{ij}(t^*)|}{a_i(t^*)}\times|f_j(\int_{t^*-\tau_{ij}}^{t^*}t_{ij}(t^*-s)y_j(s)\mathrm{d}s)|+(1-\mu)\times\frac{|I_i(t^*)|}{a_i(t^*)}\leqslant\sum_{j=1}^{p}m_{i,n+j}dh_{n+j}+D_i$. 这与$dh_i>\sum_{j=1}^{n+p}m_{ij}dh_j+D_i, i=1,2,\cdots,n$,矛盾. 从而$|\psi(u,\mu)|_0>0$成立. 又$\psi(u,\mu)\neq 0, u\in\partial\Omega\cap\mathrm{Ker}L$. 利用拓扑度的性质,且取$J$为恒等映射$I:\mathrm{Im}Q\to\mathrm{Ker}L$,则$\deg(JQN,\Omega\cap\mathrm{Ker}L,0)=\deg(\psi(\cdot,0),\Omega\cap\mathrm{Ker}L,0)=\deg(\psi(\cdot,1),\Omega\cap\mathrm{Ker}L,0)=\deg(\mathrm{diag}(-\bar{a}_1,\cdots,-\bar{a}_n,-\bar{b}_1,\cdots,-\bar{b}_p),\Omega\cap\mathrm{Ker}L,0)=(-1)^{n+p}$.

根据Gaines和Mawhin定理[5]可知,系统(1)至少存在一个ω-周期解.

定理2 假设对所有的$t,x,y\in R$,存在非负常数p_j和q_i使$|f_j(x)-f_j(y)|\leqslant p_j|x-y|$, $|g_i(x)-g_i(y)|\leqslant q_i|x-y|$, $i=1,2,\cdots,n, j=1,2,\cdots,p$. $\rho(M)<1$, M为(A2)中的定义. 则系统(1)存在唯一的ω-周期解,并且当$n\to\infty$时,其它的解都指数收敛于它,且收敛率λ满足

$$\max\left\{\frac{a_0\int_0^{\tau_{ij}}t_{ij}(u)e^{\lambda u}\mathrm{d}u}{a_0-\lambda},\frac{b_0\int_0^{\sigma_{ji}}\gamma_{ji}(u)e^{\lambda u}\mathrm{d}u}{b_0-\lambda}\right\}\rho(M)<1 \tag{9}$$

其中,$a_0 = \min\limits_{1\leqslant i\leqslant n}\{[a_i(t)]^l\}, b_0 = \min\limits_{1\leqslant j\leqslant p}\{[b_j(t)]^+\}$.

证明:由假设得 $|f_j(x)| \leqslant p_j|x| + |f_j(0)|$ 和 $|g_i(x)| \leqslant q_i|x| + |g_i(0)|$. 假设 (A1) 成立,且 $\alpha_j = [f_j(0)]^+, \beta_i = [g_i(0)]^+$ 由定理1知 (1) 至少存在一个 ω 周期解. 令 $(x(t), y(t))^{\mathrm{T}}$ 是系统 (1) 的任意一个解,令 $u(t) = x(t) - x^*(t), v(t) = y(t) - y^*(t)$,则有

$$\begin{cases}\dot{u}_i(t) = -a_i(t)u_i(t) + \sum\limits_{j=1}^{p} a_{ij}(t)F_j, i = 1,2,\cdots,n \\ \dot{v}_j(t) = -b_j(t)v_j(t) + \sum\limits_{i=1}^{n} b_{ji}(t)G_i, j = 1,2,\cdots,p\end{cases} \tag{10}$$

$$F_j = f_j\left(\int_{t-\tau_{ij}}^{t} t_{ij}(t-s)y_j(s)\mathrm{d}s\right) - f_j\left(\int_{t-\tau_{ij}}^{t} t_{ij}(t-s)y_j^*(s)\mathrm{d}s\right)$$

和 $G_i = g_i\left(\int_{t-\sigma_{ji}}^{t}\gamma_{ji}(t-s)x_i(s)\mathrm{d}s\right) - g_j\left(\int_{t-\sigma_{ji}}^{t}\gamma_{ji}(t-s)x_i^*(s)\mathrm{d}s\right)$. 只需证系统 (10) 的平衡点 $(0,0,\cdots,0)^{\mathrm{T}}$ 是全局指数稳定的. 由文献[6]知 $\sum\limits_{j=1}^{n+p} d_i^{-1}d_j m_{ij} < 1, i = 1,2,\cdots,n+p$. 存在一个充分小的常数 $\lambda \in (0, \min\{a_0, b_0\})$ 使

$$\max_{1\leqslant i\leqslant n,1\leqslant j\leqslant p}\left\{\frac{[a_i(t)]^l\int_0^{\tau_{ij}} t_{ij}(u)e^{\lambda u}du}{[a_i(t)]^l - \lambda}\sum_{k=1}^{n+p} d_i^{-1}d_k m_{ik},\right.$$

$$\left.\frac{[b_j(t)]^l\int_0^{\sigma_{ji}}\gamma_{ji}(u)e^{\lambda u}du}{[b_j(t)]^l - \lambda}\sum_{k=1}^{n+p} d_{n+j}^{-1}d_k m_{n+j,k}\right\} < 1 \tag{11}$$

令 $(u(t), v(t))^{\mathrm{T}}$ 是系统 (10) 具初始条件 $\Phi = (\varphi\psi)^{\mathrm{T}} \in C(K)$ 的解. 当 $t \geqslant 0$ 时,令 $U_i(t) = d_i^{-1}|u_i(t)|e^{\lambda t}$,否则,令 $U_i(t) = d_i^{-1}|u_i(t)|$,且当 $t \geqslant 0$,令 $V_j(t) = d_{n+j}^{-1}|v_j(t)|e^{\lambda t}$,否则 $U_i(t) = d_{n+j}^{-1}|v_j(t)|$. 于是,由 (10) 可得,对 $t \geqslant 0, i = 1, 2,\cdots,n, j = 1,2,\cdots,p$,有

$$|U_i(t)| \leqslant \exp\left[\int_t^0 (a_i(s) - \lambda)\mathrm{d}s\right]|U_i(0)| + \sum_{j=1}^{p} d_i^{-1}d_{n+j}\int_0^{\tau_{ij}} t_{ij}(u)e^{\lambda u}\mathrm{d}u\int_0^t \exp\left[\int_t^s (a_i(\theta) - \lambda)\mathrm{d}\theta\right] \times |a_{ij}(s)||p_j|\|V_j(s)\|_\tau \mathrm{d}s,$$

$$|V_j(t)| \leqslant \exp\left[\int_t^0 (b_j(s) - \lambda)\mathrm{d}s\right]|V_i(0)| + \sum_{i=1}^{n} d_{n+j}^{-1}d_i\int_0^{\sigma_{ji}}\gamma_{ji}(u)e^{\lambda u}\mathrm{d}u\int_0^t \exp\left[\int_t^s (b_j(\theta) - \lambda)\mathrm{d}\theta\right] \times |b_{ji}(s)||q_i|\|U_i(s)\|_\sigma \mathrm{d}s \tag{12}$$

其中 $\|U_i(t)\|_\sigma = \sup\limits_{s\in[t-\sigma,t]}\{|U_i(s)|\}$, $\|V_j(t)\|_\tau = \sup\limits_{s\in[t-\tau,t]}\{|V_j(s)|\}$. 记 $\overline{m} = \max\limits_{1\leqslant i\leqslant n,1\leqslant j\leqslant p}\{d_i^{-1}\|\varphi_i\|_\sigma, d_{n+j}^{-1}\|\psi_j\|_\tau\}$,假设存在某个 $t_1 \geqslant 0$ 和 $k \in \{1,2,\cdots,n\}$ 使 $U_k(t_1) = \overline{m} + \varepsilon$, $U_i(t) \leqslant \overline{m} + \varepsilon, V_j(t) \leqslant \overline{m} + \varepsilon, t \in [0, t_1), i \in \{1, 2,\cdots,n\}, j \in \{1,2,\cdots,p\}$. 由式 (12) 得

$$\overline{m} + \varepsilon = U_k(t_1) \leqslant \exp\left[\int_{t_1}^0 (a_k(s) - \lambda)\mathrm{d}s\right] \times U_k(0) + \sum_{j=1}^{p} d_k^{-1}d_{n+j}\int_0^{\tau_{ij}} t_{ij}(u)e^{\lambda u}\mathrm{d}u\int_0^{t_1}\exp\left[\int_{t_1}^s (a_k(\theta) - \lambda)\mathrm{d}\theta\right]|a_{kj}(s)|p_j\|V_j(s)\|_\tau \mathrm{d}s < (\overline{m} + \varepsilon) \times \exp\left[\int_{t_1}^0 (a_k(s) - \lambda)\mathrm{d}s\right] + (\overline{m} + \varepsilon) + \left\{1 - \exp\left[\int_{t_1}^0 (a_k(s) - \lambda)\mathrm{d}s\right]\right\} = \overline{m} + \varepsilon$$

矛盾. 故对任意充分小的常数 $\varepsilon > 0, t \geqslant 0$,有 $U_i(t) < \overline{m} + \varepsilon, V_j(t) < \overline{m} + \varepsilon$,当 $\varepsilon \to 0$. 时,$U_i(t) \leqslant \overline{m}, V_j(t) \leqslant \overline{m}$,又 $U_i(t) \leqslant d_r^{-1}\|\varphi_r\|_\sigma, V_j(t) \leqslant d_r^{-1}\|\varphi_r\|_\sigma$ 即 $\exists \overline{M} > 1$ 使 $|(U(t), V(t))^{\mathrm{T}}| \leqslant \overline{M}\|\Phi\|, t \geqslant 0$. 因此存在某个常数 $m \geqslant 1$ 使得 $|(u(t), v(t))^{\mathrm{T}}| \leqslant m\|\Phi\|e^{-\lambda t}, t \geqslant 0$ 因此系统 (10) 的平衡点 $(0,0,\cdots,0)^{\mathrm{T}}$ 是全局稳定的.

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